(multipoint- and Baker–Gammel approximants, Padé–Laurent, Padé–Fourier, Padé–Tchebycheff, Laurent–Padé, multivariate approximants) and Chapter 8 (155 pages) on multiseries (simultaneous approximation, operator approximants, vector-Padé, integral and algebraic approximants). For anyone interested in the subject, this is the heart of the book: an up-to-date account of the existing directions of research on Padé–Hermite approximation; after reading this book one has only to check the reviews since the end of 1996 to find out where the research is today.

The book then concludes with three chapters on applications: the connection with integral equations and quantum mechanics (58 pages), the connection with numerical analysis (46 pages, a.o. Crank–Nicholson and Carathéodory–Fejér), and the connection with quantum field theory (16 pages). Included are also an appendix containing a Fortran FUNCTION call to calculate approximants when a section of the power series is given (less than 200 lines of code), a bibliography (46 pages!), and a short index.

There can only be one conclusion: this book is indispensable to the researcher and the would-be researcher in the field of Padé approximation. Comparing this edition to the first one (in two volumes, 1981) one can only feel great admiration for the authors: they have done a splendid job incorporating all major developments in the theory and application of Padé approximation over the latest 15 years!

It is to be hoped that the reader will treat the examples in this volume as *exercises* (which they were originally in the first edition) that have to be worked: only through hands-on experience can one hope to master the subject.

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S. L. Sobolev and L. Vaskevich, *The Theory of Cubature Formulas*, Mathematics and Its Applications **415**, Kluwer, Dordrecht, 1997, xxi + 416 pp.

The following books dealing with numerical integration in higher dimensions are well known: Davis and Rabinowitz [1], Engels [2], Mysovskikh [3], Sloan and Joe [4], and Stroud [7]. They mainly consider constructive algebraic and number-theoretic topics of cubature. So it is not surprising that this book by Vaskevich and Sobolev on the functional-analytic aspects of cubature mainly presents material which is not covered in the works mentioned above.

Sobolev's first book on cubature [5] appeared in 1974. The translation of the Russian title is *Introduction to Cubature Formulas*. The book mainly touches on functional analysis and gives a new outlook on applications in numerical integration. An English translation [6] was published in 1992 under the title *Cubature Formulas and Modern Analysis*. In this edition the functional analytic part is abridged while the part on cubature is enlarged. Unfortunately the index of notation is dropped, making the book uncomfortable for readers not acquainted with the notation.

This book is based on these books, but is written and enlarged in a different manner. The main point is Sobolev's view of cubature and the new results following from his works. Since fundamental tools from functional analysis and partial differential equations are applied, the reader should have some knowledge of these fields.

Chapter 1 sketches the main problems and methods that are discussed in the sequel and, in particular, Sobolev's main ideas are presented. The following concept is basic. For φ in a Banach-space $\mathscr{B}$ the integral

$$\mathscr{I}(\varphi) = \int_\Omega \varphi(x)\,dx = \int \chi_\Omega(x)\,\varphi(x)\,dx$$

is to be approximated by

$$\mathscr{I}^*(\varphi) = \sum_{k=1}^{N} c_k \varphi(x^{(k)}) = \int \sum_{k=1}^{N} c_k \underbrace{\delta(x - x^{(k)})}_{\in \mathscr{B}^*}\,dx,$$

where δ denotes the Dirac delta function. The error can be written as

$$(l, \varphi) = \mathscr{I}(\varphi) - \mathscr{I}^*(\varphi) = \int \underbrace{\left[\chi_\Omega(x) - \sum_{k=1}^{N} c_k \delta(x - x^{(k)})\right]}_{l(x) \in \mathscr{B}^*} \varphi(x)\,dx.$$

For a suitable space $\mathscr{B}$, embedded in $\mathscr{C}(\Omega)$, the norm of the error

$$\|l(x)\|_{\mathscr{B}^*} = \sup_{\|\varphi\|_{\mathscr{B}} = 1} |(l, \varphi)|$$

is studied in order to determine optimal formulas based on a lattice of nodes, restricted to the domain of integration. The lattice-points can be written as $x^{(\gamma)} = hH\gamma$, where γ is an integer vector, H a lattice matrix ($\det H = 1$), and h the mesh-size. In many cases an explicit form of the error can be obtained by constructing $\mathscr{B}$-extremal functions. This will be studied first of all in the spaces $L_2^{(m)}$, $\tilde{L}_2^{(m)}$, $L_2^{(m)}(\Omega)$, and, $W_2^{(m)}$, $\tilde{W}_2^{(m)}$, $W_2^{(m)}(\Omega)$.

Before discussing this in detail, a completely different topic is presented in Chapter 2, *Cubature Formulas of Finite Order*. Here Sobolev studies invariant formulas and further refinements are gathered. The nodes of these formulas are invariant with respect to a group G of rotations and have a prefixed finite-dimensional space of exactness. In order to construct such formulas it is sufficient to consider the G-invariant elements of that space.

The concept of an error with a regular boundary layer is central for asymptotically optimal formulas. This is discussed in Chapters 3–5, *Formulas with Regular Boundary Layer for Rational Polyhedra*, *The Rate of Convergence of Cubature Formulas*, and *Cubature Formulas with Regular Boundary Layer*. By regarding different layers in the lattice of nodes it is possible to decompose the error in order to find asymptotically optimal formulas. Formulas with a regular boundary layer are constructed and an $L_2^{(m)}$-asymptotic expansion of the norm of the error is given. Further generalizations by Polovinkin follow in Chapter 6, *Universal Asymptotic Optimality*.

In Chapter 7, *Cubature Formulas of Infinite Order*, error estimates for formulas of infinite order are studied. Such formulas have an error $O(h^m)$ for all integers m. They are studied in $L_2^{(m)}$ and some special classes of functions.

Chapters 8–9, *Functions of a Discrete Variable and Optimal Formulas*, are devoted to $L_2^{(m)}$-optimal lattices; in particular algorithms to obtain such formulas are given. The book finishes with 307 references and notation and subject indexes. While the latter are useful, the references are not balanced. The overview on papers in Russian is interesting and seems to be complete concerning the topics covered in the book. Other references, however, have no direct connection to the text, they are compiled eclectically without being exhaustive to any special subject of cubature. Several misprints might be a problem for beginners and nonspecialists. Nevertheless, the book is a must for researchers in the field of cubature and of interest for everybody who wants to get information on methods for numerical integration. It complements the books on multidimensional numerical integration mentioned at the beginning.

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H. Dette and W. J. Studden, *The Theory of Canonical Moments with Applications in Statistics, Probability, and Analysis*, Wiley Series in Probability and Statistics, Wiley, New York, 1997, xvii + 327 pp.

This monograph is concerned with the unfamiliar, and under appreciated, theory and applications of canonical moments of a probability measure μ on an interval $[a, b]$ of the real line and measures on the circle. Roughly speaking, the canonical moments successively define the "relative position" of the ordinary moments c_k, given $c_1, ..., c_{k-1}$ in the set of all ordinary moment sequences. Canonical moments seem to be more intimately or intrinsically related to the measure μ than the ordinary moments. Furthermore, they are invariant under linear transformations of the measure. In this book it is shown that it is often easier to describe measures in terms of canonical moments than ordinary moments and that they have other simple interesting properties. The authors also show how these moments can be very useful in problems of design of experiments, birth and death chains, and approximation theory.

Canonical moments arc related to orthogonal polynomials and continued fractions. For a probability measure μ on the interval $[0, 1]$ we have the following relation between the canonical moments p_k and the Stieltjes transform of μ

$$\int_0^1 \frac{d\mu(x)}{z-x} = \frac{1|}{|z} - \frac{p_1|}{|1} - \frac{(1-p_1)\, p_2|}{|z} - \frac{(1-p_2)\, p_3|}{|1} - \cdots, \qquad z \notin [0, 1].$$

The first three chapters of the book provide the theoretical background. Chapter 9, with both theory and applications, is devoted to canonical moments for the circle or trigonometric functions. The other chapters contain various applications. One of the major applications is the determination of optimal designs for polynomial regression, which is presented in Chapters 5 and 6.

A special chapter, Chapter 7, is devoted to applications in approximation theory. It is illustrated that they can be used for deriving the asymptotic zero distribution of classical orthogonal polynomials. Another application